A simulation model of a long-distance passenger rail service

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CPR CEEC 2014
Prague, 19. 6. 2014

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Literature

Two computer simulation models

- PRAISE model (Preston et al. 1999, Whelan & Johnson 2004, Johnson & Nash 2012)
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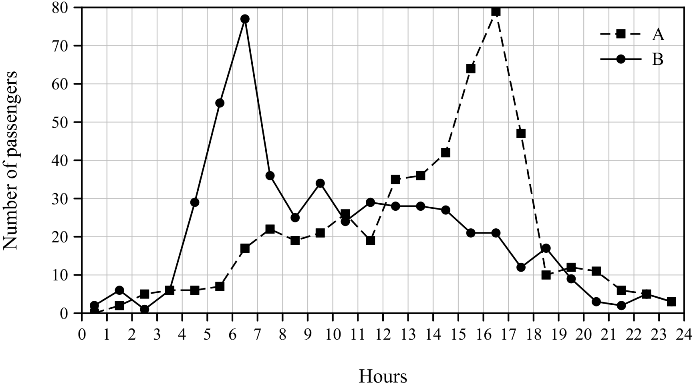

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Infrastructure/technology constraints:

- trains are allowed to depart every r minutes
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Profit of a TOC:

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- entry

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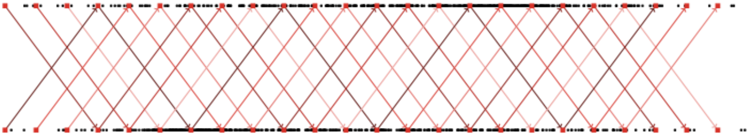

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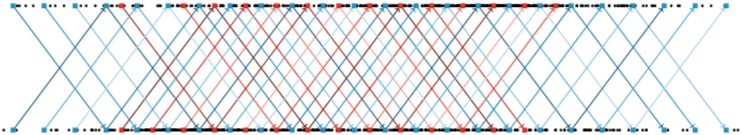

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Adjusting fares

Each period t

TOC chooses the fare

- p_{t-1}
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in order to maximize its profit Π_t.

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In period $t = e^*$ of each entry cycle

Each train in a random order chooses the departure time

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In period 1 of each entry cycle

The TOC eliminates departures using one of the options:

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Simulations – monopoly market

Simulations for parameters:

- fare-adjusting phase $T_P = 100$ periods
- exit cycle $E = 5$ periods (departure time adjustment $e^* = 2$)
- reservation price $p^R = 200$
- price-adjustment $\epsilon_t \sim U(0, 1]$
- per-minute schedule-delay cost $w = 1/60$
- the simulation ends after $f = 100$ cycles
- operational cost of a trip $C_j = 2{,}000$
- daily cost of a train $C_t = \{7{,}000; 10{,}000; 13{,}000\}$
- random passenger distributions RI = 1, 2, 3
- random-seed 1, 2, 3,. . ., 2000

Total number of simulations is 18,000 (Metacentrum).

Results

C_t	RI	S	trains m	depart. n	profit π	price p
7,000	1	4	6	28	59,027 (0.56)	175 (0.0006)
	2	43	6	28	57,411 (2.36)	175 (0.0027)
	3	45	6	28	59,257 (2.54)	175 (0.0028)
10,000	1	5	5	24	41,693 (4.3)	175 (0.005)
	2	40	5	24	42,185 (4.1)	175 (0.005)
	3	36	5	22	42,044 (3.1)	175 (0.004)
13,000	1	457	3	12	29,281 (22.3)	175 (0.10)
	2	102	3	14	32,548 (16.6)	173 (1.22)
	3	106	3	14	29,804 (4.9)	173 (0.03)

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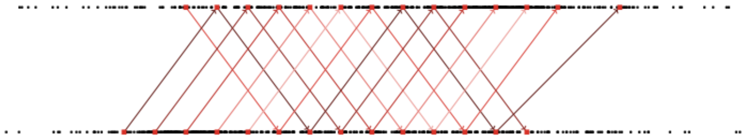

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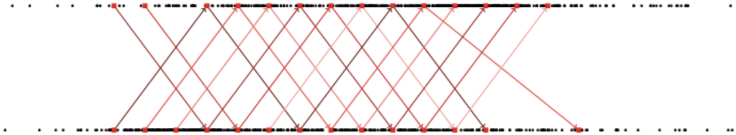

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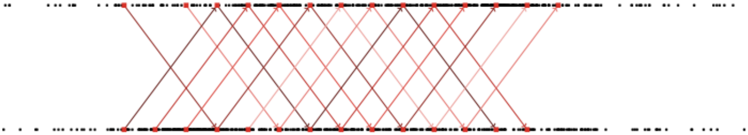

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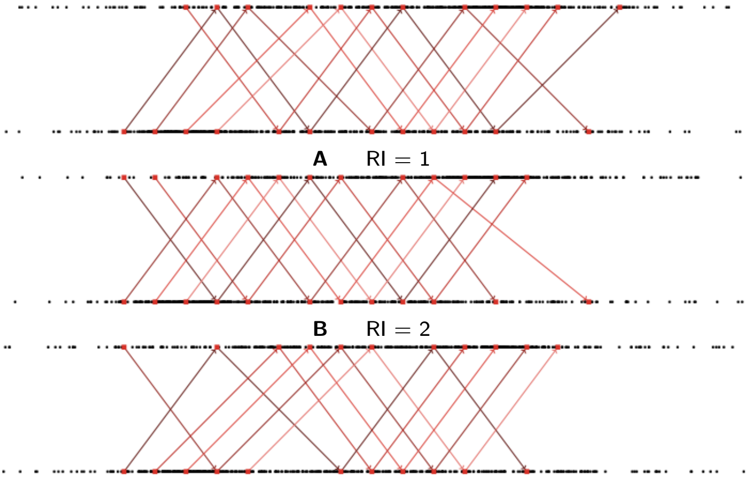

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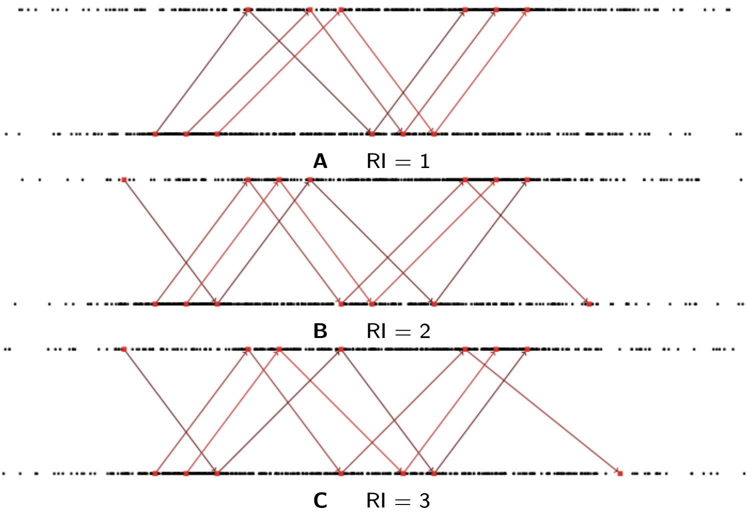

Simulations – entry

Simulations for parameters:

- random passenger distributions with random seed RI $= 3$ and a daily cost of a train $C_t^I = \{7{,}000; 10{,}000; 13{,}000\}$
- exit cycle $E = 50$ periods (departure time adjust. $e^* = 10$)
- price-adjustment $\epsilon_t \sim U(0, 2]$
- daily cost of a train $C_t^E = \{C_t^I; C_t^I - 3{,}000; C_t^I - 6{,}000\}$
- random-seed 1, 2, 3,. . ., 2000

Total number of simulations is 18,000 (Metacentrum).

Results

C^I_t	C^E_t	m^E	n^E	p^E	p^I	π^E
7,000	1,000	2.2 (0.4)	4.6 (1)	128 (10)	174 (1)	31,160 (932)
	4,000	1.7 (0.5)	4.2 (0.6)	121 (13)	174 (0.7)	24,643 (570)
	7,000	1 (0.2)	4.4 (0.8)	106 (6)	174 (1.2)	20,586 (601)
10,000	4000	1.32 (0.5)	3.1 (1.46)	108 (4.8)	170 (2.8)	17,974 (1,199)
	7,000	1.04 (0.2)	2.58 (1.3)	107 (6.9)	170 (1.6)	14,584 (1,232)
	10,000	1 (0)	2.5 (1)	106 (5.1)	170 (1.7)	11,566 (1,222)
13,000	7,000	1.33 (0.5)	5.1 (1.6)	112 (13)	156 (5)	8,844 (1,424)
	10,000	1.06 (0.2)	4.34 (0.95)	109 (12)	156 (4.2)	5,634 (1,586)
	13,000	1 (0)	4 (0.6)	108 (11)	156 (4)	2,731 (1,738)

Table shows show MEAN (SD) of 100 simulations with the highest profit of the entrant π_E.

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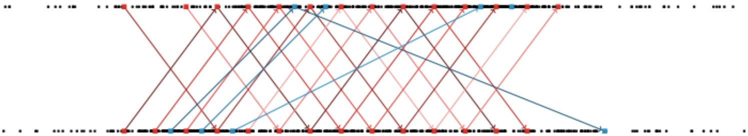
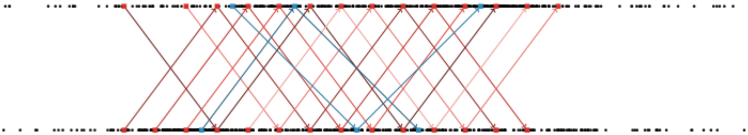
	13,000	1 (0)	4 (0.6)	108 (11)	156 (4)	2,731 (1,738)

Table shows show MEAN (SD) of 100 simulations with the highest profit of the entrant π_E.

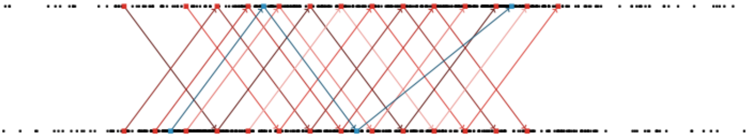
Departure times ($C_t^I = 7{,}000$)



A $C_t^E = 1{,}000$

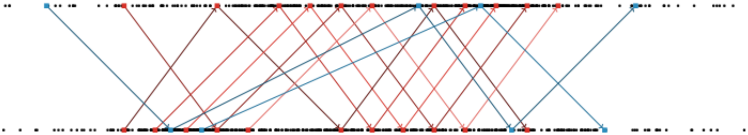


B $C_t^E = 4{,}000$

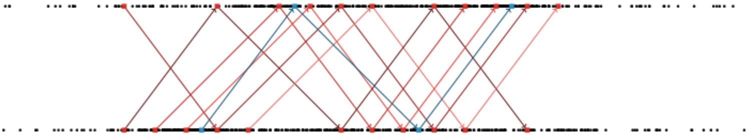


C $C_t^E = 7{,}000$

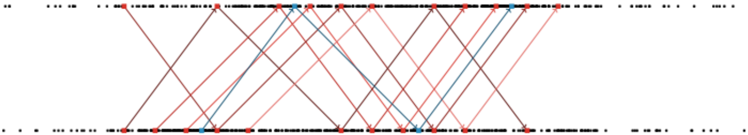
Departure times ($C_t^I = 10{,}000$)



A $C_t^E = 4{,}000$

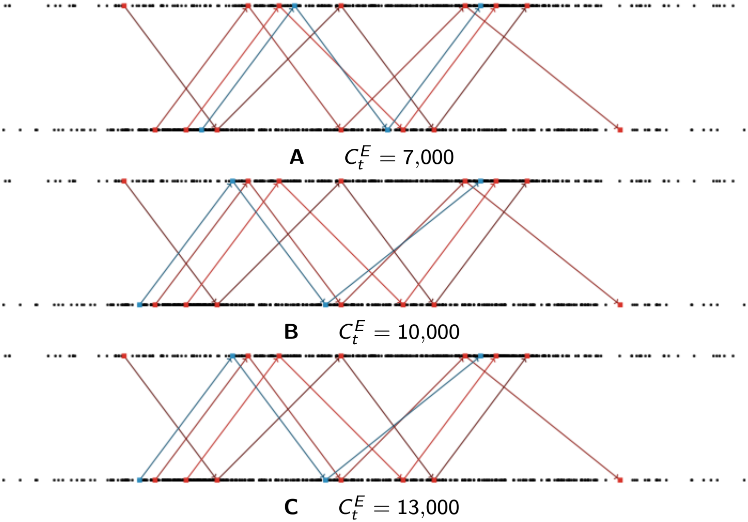


B $C_t^E = 7{,}000$



C $C_t^E = 10{,}000$

Departure times ($C_t^I = 13{,}000$)



Conclusion

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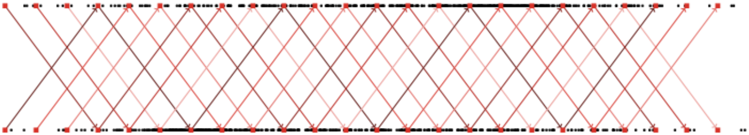
Future work

Test other algorithms (entry algorithms/genetic algorithms).

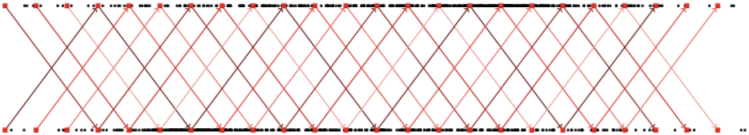
Implement local pricing.

Calibrate the model using data from Czech or Slovak markets.

Estimate the demand side of the model.



A $C_t^I = 7{,}000, \; C_t^E = 7{,}000$



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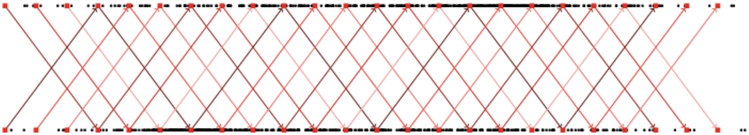
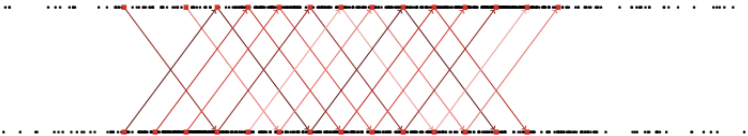


C $C_t^I = 13{,}000, \; C_t^E = 13{,}000$



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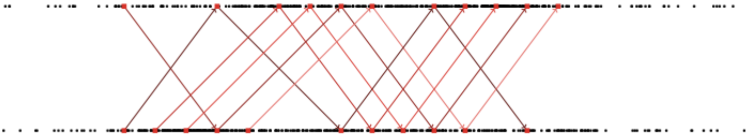

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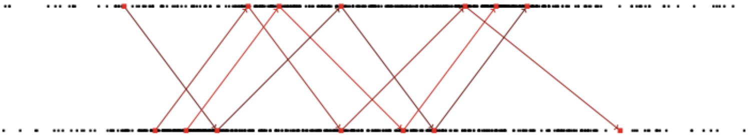
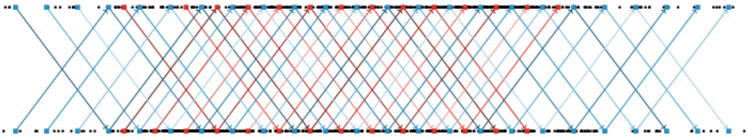


C $C_t^I = 13{,}000,\ C_t^E = 13{,}000$



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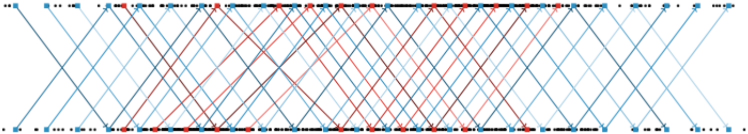

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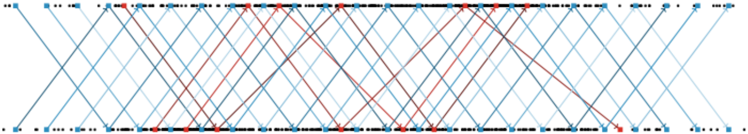

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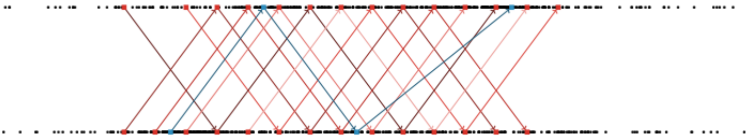

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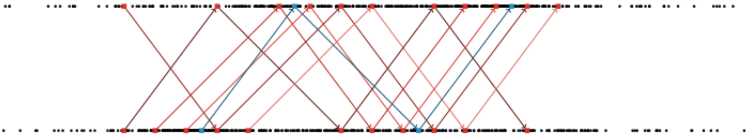

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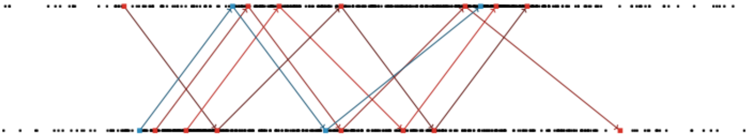

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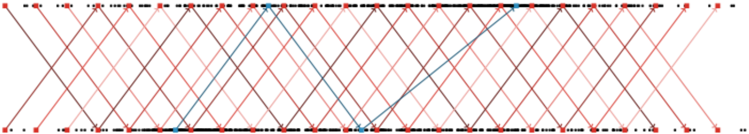

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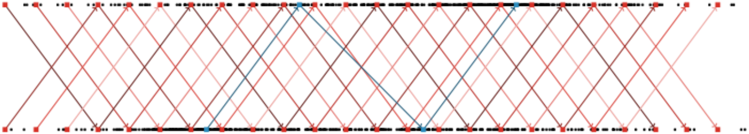

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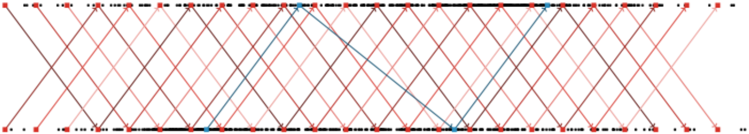
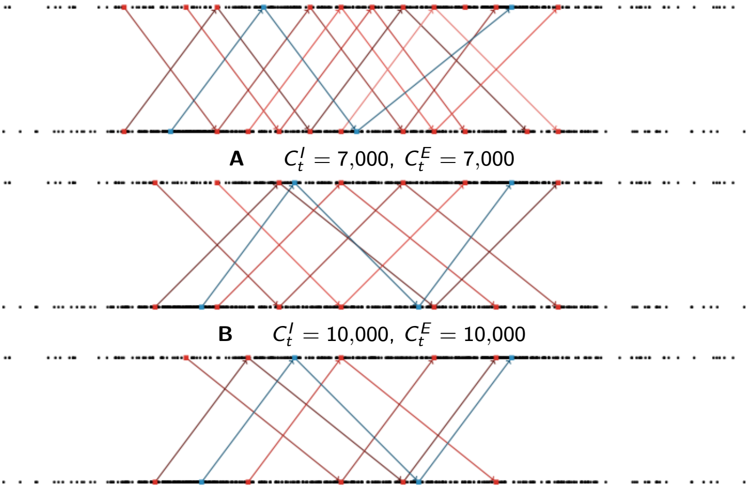


C $C_t^I = 13{,}000,\ C_t^E = 13{,}000$



A $C_t^I = 7{,}000,\ C_t^E = 7{,}000$

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